

Dense and Hot Holographic QCD: Finite Baryonic E Field

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Abstract

We investigate the response of dense and hot holographic QCD (hQCD) to a static and baryonic electric field E using the chiral model of Sakai and Sugimoto. Strong fields with $E > (\sqrt{\lambda} M_{KK})^2$ free quark pairs, causing the confined vacuum and matter state to decay. We generalize Schwinger's QED persistence function to dense hQCD. At high temperature and density, Ohm's law is derived generalizing a recent result by Karch and O'Bannon to the chiral case.

1 Introduction

The AdS/CFT approach [1] provides a framework for discussing large N_c gauge theories at strong coupling $\lambda = g_{YM}^2 N_c$. The model suggested by Sakai and Sugimoto (SS) [2] offers a holographic realization of QCD (hQCD) that has N_f flavors and is chiral. For $N_f \ll N_c$, hQCD is a gravity dual to N_f D8- $\overline{\text{D8}}$ branes embedded into a D4 background in 10 dimensions. Supersymmetry is broken by the Kaluza-Klein (KK) mechanism. The SS model yields a holographic description of pions, vectors, axials and baryons that is in good agreement with experiment [2, 3, 4]. The SS model at finite temperature has been discussed in the deconfined phase in [5] using a black-hole metric (BH). At finite baryon density and in the confined phase it has been discussed in [6, 7] using a KK metric. Isospin chemical potential and glueball have been discussed in [9].

In this paper we would like to continue our investigation of the model at baryon finite density and temperature but in the presence of a finite baryonic electric field as recently discussed by Karch and O'Bannon [10] in a non-chiral model, as a prelude to understand transport phenomena. In section 2, we briefly outline the SS model both in the confined KK metric and deconfined BH metric. In section 3, the DBI action at finite baryon density is streamlined for both the KK and BH metrics. In section 4, we discuss Ohm's law in the confined or KK metric. Above a critical value of the baryon electric field $E > E_c$ the vacuum and the dense state are unstable against quark pair creation. In section 5, we show how this pair creation translates to a vacuum persistence function thereby generalizing Schwinger's QED result to hQCD both in the vacuum and at finite density. In section 6, we derive Ohm's law in the BH background, thereby extending a recent result by Karch and Bannon [10] to the chiral case. The vacuum instability is dwarfed by thermal pair creation in the incoherent statistical averaging with a threshold value for the baryonic electric field starting at zero. Our conclusions are in section 7.

2 Sakai-Sugimoto model

In this section we summarize the Sakai-Sugimoto model (D4/D8- $\overline{\text{D8}}$ set up) for notation and completeness. For a thorough presentation we refer [2] for zero temperature and [5] for finite temperature.

At zero temperature, the confined KK metric, dilaton ϕ , and the 3-form RR field C_3 in

N_c D4-branes background are given by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (-dt^2 + \delta_{ij}dx^i dx^j + f(U)(dx^4)^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right),$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 \equiv dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4, \quad f(U) \equiv 1 - \frac{U_{\text{KK}}^3}{U^3}, \quad (1)$$

where $x^i = x^{1,2,3}$ and $U(\geq U_{\text{KK}})$ and Ω_4 are the radial coordinate and four angle variables in the $x^{5,6,7,8,9}$ direction. $R^3 \equiv \pi g_s N_c l_s^3$, with g_s and l_s the string coupling and length respectively. $V_4 = 8\pi^2/3$ is the volume of the unit S^4 and ϵ_4 is the corresponding volume form. To avoid a conical singularity at $U = U_{\text{KK}}$ the period of $\delta\tau$ of the compactified τ direction is set to $\delta\tau = \frac{4\pi}{3} \frac{R^{3/2}}{U_{\text{KK}}^{1/2}}$. The Kaluza-Klein mass is

$$M_{\text{KK}} \equiv \frac{2\pi}{\delta\tau} = \frac{3}{2} \frac{U_{\text{KK}}^{1/2}}{R^{3/2}}.$$

The parameters R , U_{KK} , and g_s may be expressed in terms of M_{KK} , $\lambda (= g_{\text{YM}} N_c)$, and l_s as

$$R^3 = \frac{1}{2} \frac{\lambda l_s^2}{M_{\text{KK}}}, \quad U_{\text{KK}} = \frac{2}{9} \lambda M_{\text{KK}} l_s^2, \quad g_s = \frac{1}{2\pi} \frac{\lambda}{M_{\text{KK}} N_c l_s}.$$

At finite temperature, there are two possibilities. One is the same as the zero temperature apart from the fact the time direction is Euclidean and compactified with a circumference $\beta = 1/T$. It corresponds to the confined phase which is the low temperature regime(KK). The other corresponds to deconfined phase of the high temperature. Its geometry contains the black hole and the pertinent BH background is

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (f(U)dt_E^2 + \delta_{ij}dx^i dx^j + (dx^4)^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right),$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 \equiv dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4, \quad f(U) \equiv 1 - \frac{U_T^3}{U^3}, \quad (2)$$

The t_E -direction must be periodic

$$\delta t_E = \frac{4\pi}{3} \left(\frac{R^3}{U_T}\right)^{1/2} \equiv \beta = \frac{1}{T} \quad (3)$$

to avoid a conical singularity. The x^4 -direction is also periodic but it has an arbitrary periodicity.

Now, consider N_f probe D8-branes in the N_c D4-branes background. With $U(N_f)$ gauge field A_M on the D8-branes, the effective action consists of the DBI action and the Chern-Simons action:

$$\begin{aligned} S_{\text{D8}} &= S_{\text{D8}}^{\text{DBI}} + S_{\text{D8}}^{\text{CS}} , \\ S_{\text{D8}}^{\text{DBI}} &= -T_8 \int d^9x \, e^{-\phi} \, \text{tr} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} , \end{aligned} \quad (4)$$

$$S_{\text{D8}}^{\text{CS}} = \frac{1}{48\pi^3} \int_{\text{D8}} C_3 \text{tr} F^3 . \quad (5)$$

where $T_8 = 1/((2\pi)^8 l_s^9)$, the tension of the D8-brane, $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$ ($M, N = 0, 1, \dots, 8$), and g_{MN} is the induced metric on D8-branes. The expressions are written for the Minkowskian metric.

3 DBI action

The induced metric on the D8 branes from the gravity background (1) and (2) may be written as

$$ds_{\text{D8}}^2 \equiv g_{tt} dt^2 + g_{xx} \delta_{ij} dx^i dx^j + g_{UU} dU^2 + g_{SS} d\Omega_4^2 \quad (6)$$

$$\equiv \alpha \left(\frac{U}{R} \right)^{3/2} dt^2 + \left(\frac{U}{R} \right)^{3/2} \delta_{ij} dx^i dx^j + \left(\frac{R}{U} \right)^{3/2} \gamma dU^2 + \left(\frac{R}{U} \right)^{3/2} U^2 d\Omega_4^2 , \quad (7)$$

where for the KK background

$$\alpha \rightarrow -1 , \quad \gamma \rightarrow \frac{1}{f(U)} + \left(\frac{\partial x^4}{\partial U} \right)^2 \left(\frac{U}{R} \right)^3 f(U) , \quad f(U) \rightarrow 1 - \left(\frac{U_{\text{KK}}}{U} \right)^3 , \quad (8)$$

and for the BH background

$$\alpha \rightarrow f(U) , \quad \gamma \rightarrow \frac{1}{f(U)} + \left(\frac{\partial x^4}{\partial U} \right)^2 \left(\frac{U}{R} \right)^3 , \quad f(U) \rightarrow 1 - \left(\frac{U_T}{U} \right)^3 . \quad (9)$$

The embedding information is encoded only in γ and thereby g_{UU} . We will use the abstract metric notations (6) to treat the confined (8) and deconfined (9) coherently in formal evaluation here. In the next section we will plug in the specific embedding and metric form.

To accommodate a static baryonic electric field on D8 branes both in vacuum and matter,

we follow [10] to define

$$A_t = A_t(U) , \quad A_x = -Et + h_x(U) . \quad (10)$$

With the induced metric (6) and the gauge fields (10) the DBI action (4) is written as

$$\begin{aligned} S_{\text{DBI}} &\equiv \int d^4x dU \mathcal{L}_{\text{DBI}} \\ &= -\mathcal{N} \int dU e^{-\phi} g_{SS}^2 g_{xx} \sqrt{|g_{tt}| g_{xx} g_{UU} - (2\pi\alpha')^2 \left(g_{xx} (A'_t)^2 + g_{UU} (\dot{A}_x)^2 - |g_{tt}| (A'_x)^2 \right)} \end{aligned} \quad (11)$$

where $\mathcal{N} \equiv (2N_f)T_8 V_4$. $2N_f$ comes from the fact that we consider N_f branes and anti-branes and $V_4 (= 8/3\pi^2)$ is the volume of the unit S^4 which is due to the trivial integral over S^4 . ' is the derivative with respect to U and $\dot{}$ is the derivative with respect to t . Since (11) is purely kinetic, the conjugate momenta D and B are conserved. Specifically,

$$D \equiv \frac{\partial \mathcal{L}_{\text{DBI}}}{\partial A'_t} = e^{-\phi} g_{SS}^2 g_{xx} \frac{-\mathcal{N}(2\pi\alpha')^2 g_{xx} A'_t}{\sqrt{|g_{tt}| g_{xx} g_{UU} - (2\pi\alpha')^2 (g_{xx} A_t'^2 + g_{UU} \dot{A}_x^2 - |g_{tt}| h_x'^2)}} \quad (12)$$

$$B \equiv \frac{\partial \mathcal{L}_{\text{DBI}}}{\partial A'_x} = e^{-\phi} g_{SS}^2 g_{xx} \frac{\mathcal{N}(2\pi\alpha')^2 |g_{tt}| h'_x}{\sqrt{|g_{tt}| g_{xx} g_{UU} - (2\pi\alpha')^2 (g_{xx} A_t'^2 + g_{UU} \dot{A}_x^2 - |g_{tt}| h_x'^2)}} \quad (13)$$

By rewriting A'_t and h'_x in terms of B , D and E , we have

$$g_{xx} A'_t(U)^2 = \frac{1}{(2\pi\alpha')^2} |g_{tt}| D^2 \frac{g_{UU} (|g_{tt}| g_{xx} - (2\pi\alpha')^2 E^2)}{\mathcal{N}^2 (2\pi\alpha')^2 |g_{tt}| g_{xx}^3 e^{-2\phi} g_{SS}^4 + |g_{tt}| D^2 - g_{xx} B^2} \quad (14)$$

$$|g_{tt}| h'_x(U)^2 = \frac{1}{(2\pi\alpha')^2} g_{xx} B^2 \frac{g_{UU} (|g_{tt}| g_{xx} - (2\pi\alpha')^2 E^2)}{\mathcal{N}^2 (2\pi\alpha')^2 |g_{tt}| g_{xx}^3 e^{-2\phi} g_{SS}^4 + |g_{tt}| D^2 - g_{xx} B^2} \quad (15)$$

The DBI action reduces to

$$S_{\text{DBI}} = -\mathcal{N} \int d^4x dU \left[e^{-2\phi} g_{SS}^4 g_{xx}^{5/2} |g_{tt}|^{1/2} g_{UU}^{1/2} \right] \sqrt{\frac{(|g_{tt}| g_{xx} - (2\pi\alpha')^2 E^2)}{|g_{tt}| g_{xx}^3 e^{-2\phi} g_{SS}^4 + \frac{|g_{tt}| D^2 - g_{xx} B^2}{\mathcal{N}^2 (2\pi\alpha')^2}}} \quad (16)$$

Notice that g_{tt}, g_{xx}, g_{SS} have nothing to do with the D8 branes embedding. They carry information of D4 branes. Only g_{UU} carries information of the $x^4(U)$. It is positive for all U . Thus the factors outside the square root are real for all U . In contrast, the argument of square root may change the sign for varying U . As we will discuss below, this change in sign is the signal of a ground state instability or decay for large E fields.

4 Ohm's law: KK

This decay is captured by a non-linear form of Ohm's law. For that, it is useful to change variable

$$U = U_0(1 + Z^2)^{1/3} , \quad (17)$$

where U_0 is the coordinate of the tip of D8- $\overline{\text{D8}}$ branes' cigar-shaped configuration, which is different from U_{KK} in general. The range of Z is $(0, \infty)$ contrary to U whose range is (U_0, ∞) . Also this range can be extended to $(-\infty, \infty)$ if we consider $\overline{\text{D8}}$ branes $(-\infty, 0)$ together with D8 branes $(0, \infty)$ in a natural way. It enables us to deal with the ADHM instanton solution in \mathbb{R}^4 [3]. It also makes the parity property of the meson fields explicit [2]. For completeness, we note the following useful relations

$$K \equiv 1 + Z^2 , \quad U = U_0 K^{1/3} , \quad dU = \frac{2U_0}{3} \frac{Z}{K^{2/3}} dZ , \quad f = 1 - \left(\frac{U_{\text{KK}}}{U_0} \right)^3 \frac{1}{K} . \quad (18)$$

From here on and for simplicity, we follow Sakai and Sugimoto [2] and choose $U_0 = U_{\text{KK}}$. The DBI action then simplifies to

$$S_{\text{DBI}} = -a \int d^4x dZ K^{1/6} \sqrt{\frac{K - \frac{b}{M_{\text{KK}}^2} E^2}{1 + \frac{D^2 - B^2}{a^2 b} K^{-5/3}}} , \quad (19)$$

where

$$a \equiv \frac{N_c N_f \lambda^3 M_{\text{KK}}^4}{3^9 \pi^5} , \quad b \equiv \frac{3^6 \pi^2}{4 \lambda^2 M_{\text{KK}}^2} . \quad (20)$$

In dense hQCD baryons are sourced by BPST instantons in bulk with a size of order $1/\sqrt{\lambda}$. They are point-like at $\lambda \rightarrow \infty$. Thus the DBI action and the matter sources read

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{DBI}} + n_B \delta(Z) A_t(Z) + \tilde{n}_B v_x \delta(Z) A_x(t, Z) , \quad (21)$$

where n_B is the baryon - anti baryon density and \tilde{n}_B is baryon + anti baryon density. The first source contribution is that of static BPST instantons at $Z = 0$ as initially discussed in [7]. The second term is their corresponding current with a velocity $v_x \sim 1/\lambda N_c$ with a baryon mass $M_B \sim N_c \lambda M_{\text{KK}}$. Note that we have renormalized the A_μ field here by $1/N_c$ and identified the baryon chemical potential as $A_\mu(\infty) = \mu_B - m_B$ [7].

The equations of motion are

$$D' = n_B \delta(Z) , \quad B' = \tilde{n}_B v_x \delta(Z) . \quad (22)$$

Thus

$$D = \frac{1}{2} n_B \text{sgn}(Z), \quad B = \frac{1}{2} \tilde{n}_B v_x \text{sgn}(Z) , \quad (23)$$

where $\text{sgn}(Z)$ reflects the symmetry of D8 and $\overline{\text{D8}}$ branes (chirality). We note that the conserved momenta D, B are odd functions of Z since the baryonic field A_μ is an even function of Z .

For a finite baryonic electric field E , the current contribution in (21) is seen to increase linearly with time in the action. This is expected since the static electric field pumps energy in the system. For times $t \sim M_B \sim N_c \lambda$ the present stationary (time-independent) surface analysis is flawed. This notwithstanding, the action variation with respect to A_t yields

$$\begin{aligned} \delta_{A_t} S_{\text{tot}} &= \int dZ \left[\frac{\delta \mathcal{L}}{\delta(\partial_Z A_t)} \partial_Z(\delta A_t) + n_B \delta(Z) \delta A_t(Z) \right] \\ &= \int dZ \left(\frac{1}{2} n_B \text{sgn}(Z) \partial_Z(\delta A_t) \right) + n_B \delta A_t(0) \\ &= n_B \delta A_t(\infty) . \end{aligned} \quad (24)$$

where we used the on-shell condition and $A_t(\infty) = A_t(-\infty) = \mu_B - m_B$. Note that the contribution from the source term is cancelled by the boundary contribution of the DBI action at $Z = 0$. As a result the on-shell action may be considered as a functional of $A_t(\infty)$ only and we may set $A_t(0) = 0$. Similarly for $A_x(t, 0) = 0$,

$$\delta_{A_t} S_{\text{tot}} = \tilde{n}_B v_x \delta A_x(t, \infty) . \quad (25)$$

The former is the charge, while the latter is the current. At finite density S_{tot} plays the role of the grand potential. Thus

$$\bar{S} = -a \int d^4 x dZ K^{1/6} \sqrt{\frac{K - \frac{b}{M_{\text{KK}}^2} E^2}{1 + \frac{n_B^2 - \tilde{n}_B^2 v_x^2}{4a^2 b} K^{-5/3}}} , \quad (26)$$

on shell. For $v_x = E = 0$ this result is consistent with our previous result i.e. Eq.(30) in [7] which is indeed the grand potential.

For $0 < E \leq E_c \equiv \frac{M_{KK}}{\sqrt{b}}$, $J_x (= \tilde{n}_B v_x)$ is bounded,

$$J_x < \sqrt{4a^2b + n_B^2} , \quad (27)$$

for \bar{S} to be real. For $E > E_c$, the numerator of (26) flips sign at

$$K_* = \frac{b}{M_{KK}^2} E^2 , \quad Z_* = \pm \sqrt{bE^2 - 1} . \quad (28)$$

We demand that this flip is compensated by the denominator for arbitrary v_x . Using Z_* in the denominator we get

$$\begin{aligned} J_x^2 &= 4a^2bK_*^{5/3} + n_B^2 \\ &= \frac{1}{2^{10/3}3^2\pi^{14/3}} N_f^2 N_c^2 \left(\frac{\lambda}{M_{KK}} \right)^{2/3} E^{10/3} + n_B^2 \theta(E) . \end{aligned} \quad (29)$$

In the unstable vacuum, the ensuing Ohmic's conductivity is

$$\sigma \equiv \frac{J_x}{E} = \frac{1}{2^{5/3}3\pi^{7/3}} N_c N_f \left(\frac{\lambda}{M_{KK}} \right)^{1/3} E^{2/3} . \quad (30)$$

This pair conductivity follows from quark pairs and not from baryon pairs as it scales with $N_c N_f$. E_c is strong enough to cause deconfinement of quark pairs. For $n_B \neq 0$ the second contribution in (29) is that of the baryons and anti baryons moving under the action of the *strong* electric field, with $\Delta v \sim Et/M_B \sim t/N_c$. Note that for $E = 0$, the minimum of (26) is for $v_x = 0$.

For $E > E_c$ both the vacuum with $n_B = 0$ and the dense baryonic state with $n_B \neq 0$ are unstable against pair creation of quark-antiquark states as opposed to baryon-antibaryon states. This is clearly seen from the threshold value E_c

$$E_c = \frac{M_{KK}}{\sqrt{b}} = \frac{2}{27\pi} M_{KK}^2 \lambda = \frac{54\pi M_B^2}{\lambda N_c^2} , \quad (31)$$

with $M_B = 8\pi^2 \kappa M_{KK}$ and $\kappa = \frac{\lambda N_c}{216\pi^3}$ [3] which is much smaller than M_B^2 . The baryonic electric field is strong enough to pair create quarks with *constituent masses* of order $\sqrt{\lambda} M_{KK}$ ¹.

¹It is interesting to note that in the BH background the thermal shifts of heavy quarks is $\pi\sqrt{\lambda}T/2$ with T in the unconfined phase being the analogue of M_{KK} in the confined phase.

5 Persistence Probability

The cold and dense states described by hQCD above are unstable for $E > E_c$, meaning that they decay to multiparticle states that are likely time-dependent. Following Schwinger, we will characterize this decay through its persistence probability

$$|\langle 0_+ | 0_- \rangle|^2 = e^{-2\text{Im}\bar{S}}, \quad (32)$$

where $\text{Im}\bar{S}$ is the imaginary part of the action \bar{S} (26). For finite n_B and $v_x = 0$, the action \bar{S} reads

$$\bar{S} = -a \int d^4x dZ (1 + Z^2)^{1/6} \left[\sqrt{\frac{Z^2 + 1 - \mathcal{E}^2}{1 + \mathcal{N}^2(1 + Z^2)^{-5/3}}} - \sqrt{\frac{Z^2 + 1}{1 + \mathcal{N}^2(1 + Z^2)^{-5/3}}} \right], \quad (33)$$

with $\mathcal{E}^2 \equiv \frac{b}{M_{\text{KK}}^2} E^2$, $\mathcal{N}^2 \equiv \frac{n_B^2}{4a^2b}$ and after regularizing the action by subtracting the $\mathcal{E} = 0$ contribution. E_c corresponds to $\mathcal{E}_c = 1$. For $\mathcal{E} \leq 1$ the action \bar{S} is always real, but for $\mathcal{E} > 1$ the action develops an imaginary part from the integration interval $(-Z_c, Z_c)$, where $Z_c \equiv \sqrt{\mathcal{E}^2 - 1}$. Thus

$$\text{Im}\bar{S} = \pm a \int d^4x \int_{-Z_c}^{Z_c} dZ (1 + Z^2)^{1/6} \sqrt{\frac{Z^2 + 1 - \mathcal{E}^2}{1 + \mathcal{N}^2(1 + Z^2)^{-5/3}}} \theta(\mathcal{E} - 1). \quad (34)$$

For $\mathcal{N} = 0$ the integrals unwind analytically

$$\text{Im}\bar{S} = \pm a\pi \int d^4x \left[(\mathcal{E}^2 - 1) {}_2F_1 \left(-\frac{1}{6}, \frac{1}{2}, 2, 1 - \mathcal{E}^2 \right) \theta(\mathcal{E} - 1) \right]. \quad (35)$$

where ${}_2F_1$ is the hypergeometric function and has the asymptotic behaviour as follows.

$$\begin{aligned} & {}_2F_1 \left(-\frac{1}{6}, \frac{1}{2}, 2, 1 - \mathcal{E}^2 \right) \\ & \sim 1 + \frac{1}{12}(\mathcal{E} - 1) + \frac{1}{144}(\mathcal{E} - 1)^2 + \dots \quad (\mathcal{E} \sim 1) \\ & \sim \frac{\Gamma(2/3)}{\sqrt{\pi}\Gamma(13/6)} \mathcal{E}^{1/3} + \frac{2\Gamma(-2/3)}{\sqrt{\pi}\Gamma(-1/6)} \frac{1}{\mathcal{E}} + \dots \quad (\mathcal{E} \gg 1) \end{aligned} \quad (36)$$

The persistence function is then

$$\begin{aligned}
|\langle 0_+ | 0_- \rangle|^2 &= e^{-a'(\varepsilon^2-1) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}, 2, 1-\varepsilon^2\right) \theta(\varepsilon-1)}, \\
&= 1 & (\mathcal{E} \leq 1) \\
&e^{-a'[2(\varepsilon-1)+1.17(\varepsilon-1)^2+\dots]} & (\mathcal{E} \sim 1) \\
&e^{-a'[0.71 \varepsilon^{7/3}+0.70 \varepsilon+\dots]} & (\mathcal{E} \gg 1)
\end{aligned} \tag{37}$$

with $a' \equiv a\pi \int d^4x = \frac{N_c N_f \lambda^3 M_{\text{KK}}^4}{39\pi^4} \int d^4x$, after choosing the negative sign for decay.

6 Ohm's law: BH

Since the vacuum decay under large E 's so does the coherent finite baryonic state. But what about the finite temperature problem? As finite temperature involves a statistical ensemble averaging, we may suggest that the unstable ground state is statistically irrelevant and proceed to analyse the effects of a baryonic field on the excited states (unstable by fiat) in the ensemble average. This will be checked a posteriori below.

In the BH background there are two possible gravitational configurations: 1/ a U-shaped (chirally broken phase) and 2/ a parallell-shape (chirally symmetric phase). The former yields U bounded from below by U_0 . The combination $g_{tt}g_{xx}$ has a positive minimum so the numerator is always positive for sufficiently small E . The nature of the transition which is suggestive of a metal-insulator transition [11] will be discussed elsewhere.

For high enough temperature the stable configuration is not the U-shaped configuration but the parallel configuration which is connected to the black hole. i.e. $\frac{dx^4}{dU} = 0$. Our initial instanton sources have now drowned into the BH horizon. So the ensuing analysis is the same as in the D3/D7 model [10], with the general formula of the conductivity for Dq/Dp given ((5.7) in [10]). Here and for completeness, we compute the conductivity for the parallel D8- $\overline{\text{D8}}$ branes set up in the BH background.

We only need to consider the positivity condition for the argument of square root as before. As $U \rightarrow U_T$ both the numerator and denominator are *negative* since $g_{tt} \rightarrow 0$. As $U \rightarrow \infty$ both the numerator and denominator are *positive*. So by choosing B,D,E we can choose the numerator and denominator in (16) to flip sign for the same value $U = U_*$ [10].

For the numerator

$$\begin{aligned} |g_{tt}|g_{xx}\Big|_{U=U_*} &= (2\pi\alpha')^2 E^2 \\ \Rightarrow U_* &= (U_T^3 + R^3(2\pi\alpha'E)^2)^{1/3} . \end{aligned} \quad (38)$$

Inserting this value of U_* in the denominator yields the induced current

$$\begin{aligned} J_x^2 &= \left(\mathcal{N}^2(2\pi\alpha')^2 |g_{tt}|g_{xx}^2 e^{-2\phi} g_{SS}^4 + \frac{|g_{tt}|}{g_{xx}} J_t^2 \right) \Big|_{U=U_*} \\ &= \left(\frac{\mathcal{N}^2(2\pi\alpha')^4 R^6}{g_s^2} (U_T^3 + R^3(2\pi\alpha'E)^2)^{2/3} + \frac{(2\pi\alpha')^2}{\frac{U_T^3}{R^3} + (2\pi\alpha'E)^2} J_t^2 \right) E^2 , \end{aligned} \quad (39)$$

where $J_x = B$ and $J_t = D (= n_B)$ are now defined as in [10]. Setting $U_T = \frac{16\pi^2}{9} T^2 R^3$, $\lambda = g_s N_c$ yield the Ohmic conductivity for the chiral SS model

$$\sigma = \frac{J_x}{E} = \sqrt{\left(\frac{4l_s N_f N_c \lambda T^2}{27} \right)^2 (1 + e^2)^{2/3} + \frac{d^2}{1 + e^2}} , \quad (40)$$

where

$$e \equiv \frac{3^3 E}{2^5 \pi^3 T^3 \lambda l_s} , \quad d \equiv \frac{3^3 J_t}{2^5 \pi^3 T^3 \lambda l_s} , \quad (41)$$

which is consistent with the result in [10] for massless but non-chiral quarks. The induced thermal current sets in for any $E \geq 0$ (large or small) with a conductivity σ of order $N_c N_f \lambda T^2 l_s$ at high temperature, which involves only thermal pairs with zero threshold for E . It dwarfs the induced vacuum pairs by a factor of $\lambda^{2/3}$. The unstable vacuum state is statistically irrelevant. This is not the case at $T = 0$ and/or very large baryonic densities.

7 Conclusions

We have extended our recent holographic analysis of the SS model at finite density, to the case of finite temperature and finite baryonic electric field. For $E > E_c$ the stationary SS ground state breaks down by quark pair creation. This phenomenon permeates both the cold and hot states of hQCD. The vacuum persistence probability is derived, generalizing Schwinger's QED result to hQCD. At finite temperature, the baryonic electric field yields a thermal conductivity at finite temperature and density that is a direct generalization of

Karch and Bannon’s Ohm’s law in the chiral model. We have argued that the vacuum instability is statistically irrelevant in hot hQCD.

8 Acknowledgments

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9 Note added

While typing our results, a recent analysis appeared in [12] that addresses similar issues in the model with cusp surfaces at $L \neq \pi$. Our results are all for cusplless surfaces with $L = \pi$ for the KK background, and the parallel or deconfined configuration for the BH background.

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